Coupled Nonlinear Transistor Microwave Oscillators

Comprehensive Examination

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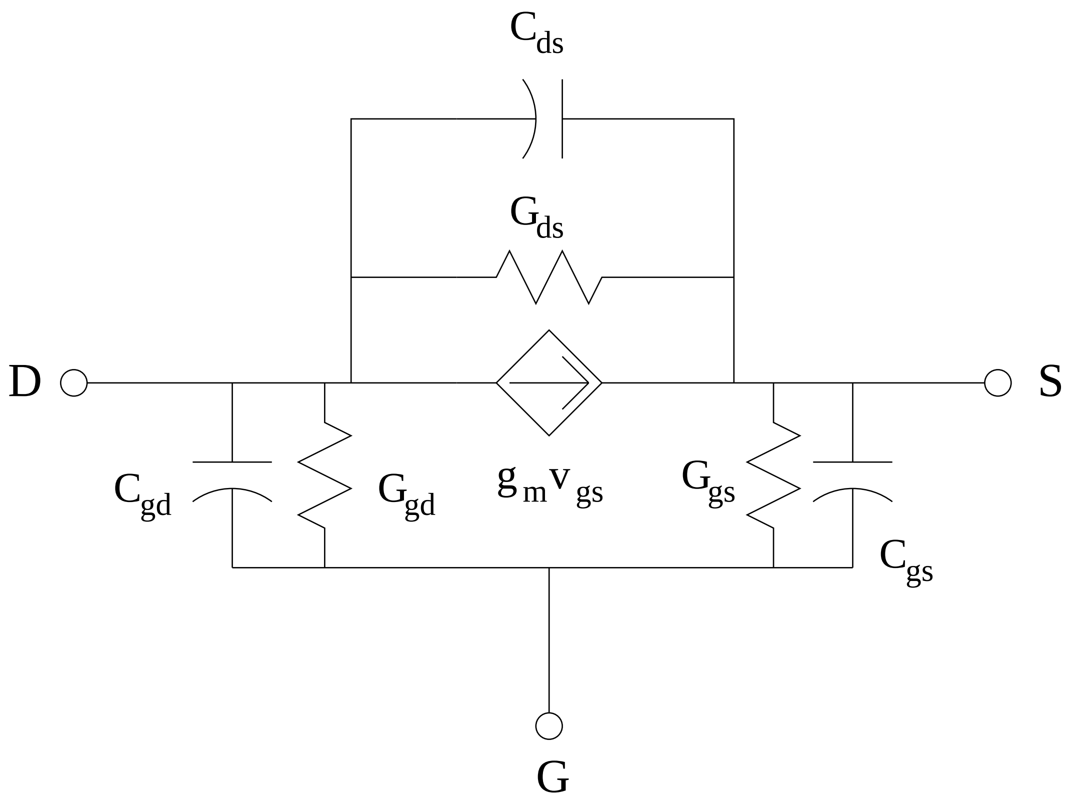
Introduction

- Types of quasi-optical oscillators
- Typical output of grid oscillator
- Reason for use of quasi-optical techniques

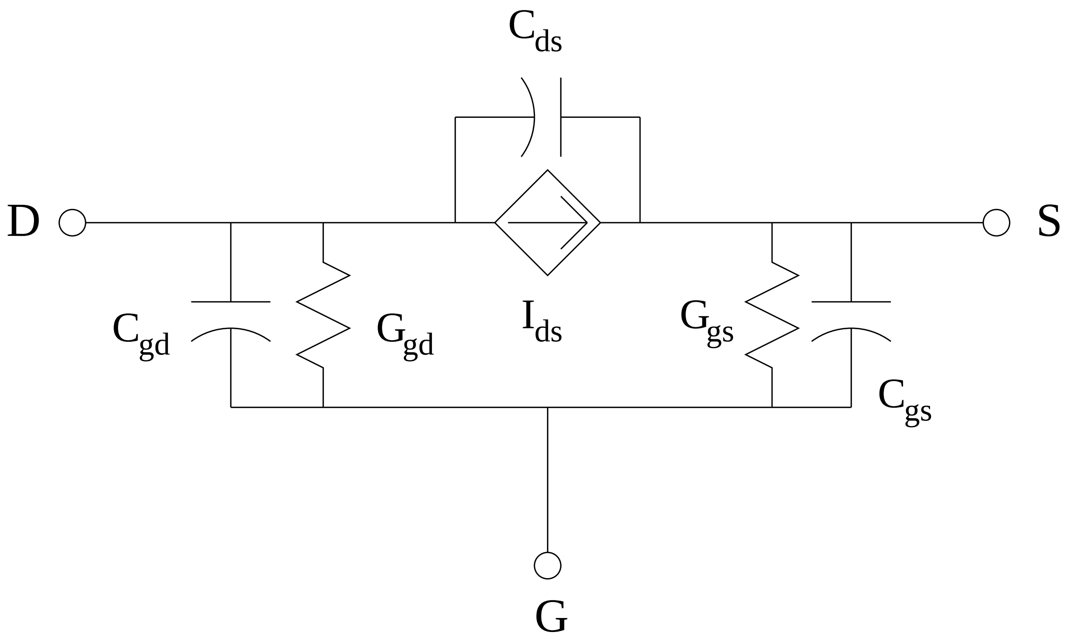
Justification for Work

- Understand locking behavior of multiple oscillators
- Describe behavior of large scale quasi-optical oscillator power combiners
- Difficulty in making detailed measurements of oscillator operation

Linear Transistor Model



Curtice Cubic Transistor Model



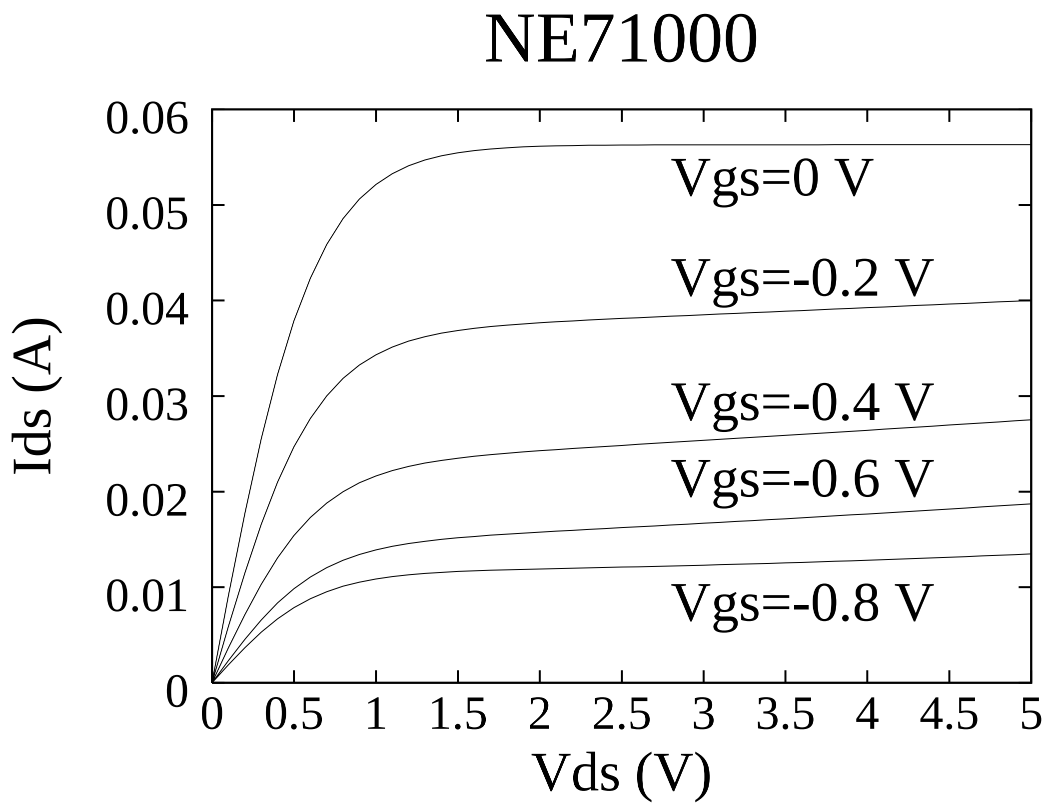
$$I_{ds} = \left(A_0 + A_1 v_1 + A_2 v_1^2 + A_3 v_1^3\right) \tanh\left(\gamma v_{ds}\right)$$

$$v_1 = v_{gs}\left[1 + \beta\left(v_{ds0} - v_{ds}\right)\right]$$

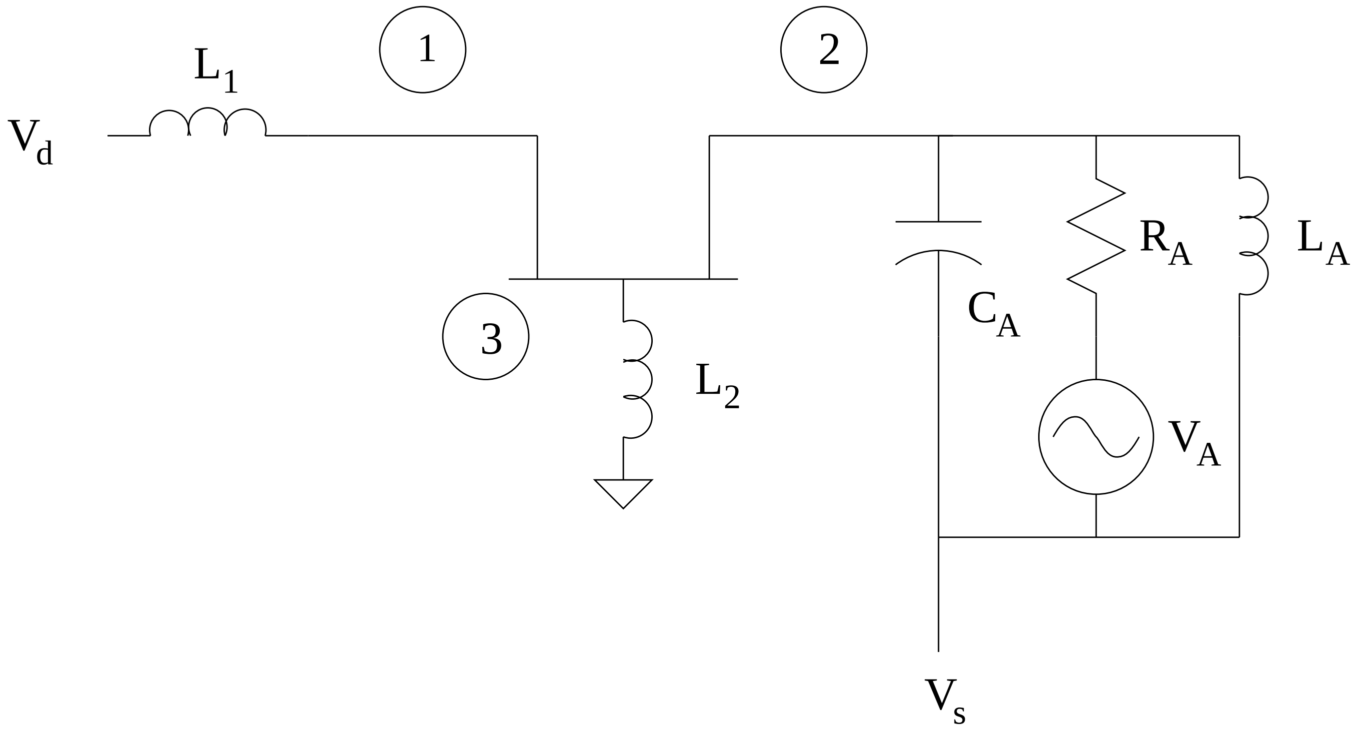
$$C_d = C_{d0}\left(1 - \frac{v_{applied}}{v_{bi}}\right)^{-1/2}$$

$$G_d = \frac{I_d}{V_d} = \frac{I_s}{V_d}\left(e^{V_d/nV_t} - 1\right)$$

Model I-V Curves



Source-Load Circuit Model



$$
\begin{aligned}
0 &= \frac{1}{L_1}\int (v_1 - V_d)\,dt + C_{gd}(\dot{v}_1 - \dot{v}_3) + G_{gd}(v_1 - v_3) + C_{ds}(\dot{v}_1 - \dot{v}_2) + I_{ds} \\
0 &= C_{ds}(\dot{v}_2 - \dot{v}_1) - I_{ds} + C_{gs}(\dot{v}_2 - \dot{v}_3) + G_{gs}(v_2 - v_3) + C_A\dot{v}_2 + \frac{v_2 - v_A - V_s}{R_A} + \frac{1}{L_A}\int (v_2 - V_s)\,dt \\
0 &= \frac{1}{L_2}\int v_3 dt + C_{gd}(\dot{v}_3 - \dot{v}_1) + G_{gd}(v_3 - v_1) + C_{gs}(\dot{v}_3 - \dot{v}_2) + G_{gs}(v_3 - v_2)
\end{aligned}
$$

Resulting Equations

$$\left.\begin{array}{rcl}
\dot{u}_1 &=& u_4 \\
\dot{u}_2 &=& u_5 \\
\dot{u}_3 &=& u_6 \\
\dot{u}_4 &=& \dfrac{k_1\left(f + C_{gd}h'\right)}{k_2} \\
\dot{u}_5 &=& \dfrac{C_{gd}}{k_2}\left(f + C_{gd}h'\right) + h' \\
\dot{u}_6 &=& \dfrac{1}{C_A}\left(f + g + h\right)
\end{array}\right\}$$

where

$$u_1 = v_1 - v_2 = v_{ds},\ u_2 = v_3 - v_2 = v_{gs},\ u_3 = v_2 - V_s,\ u_4 = \dot{u}_1,\ u_5 = \dot{u}_2, \text{ and } u_6 = \dot{u}_3$$

and

$$f = -\dot{I}_{ds} - \frac{1}{L_1}\left(u_1 + u_3 + v_s - v_d\right) - \left(\dot{C}_{gd} + G_{gd}\right)\left(\dot{u}_1 - \dot{u}_2\right) - \dot{G}_{gd}\left(u_1 - u_2\right)$$

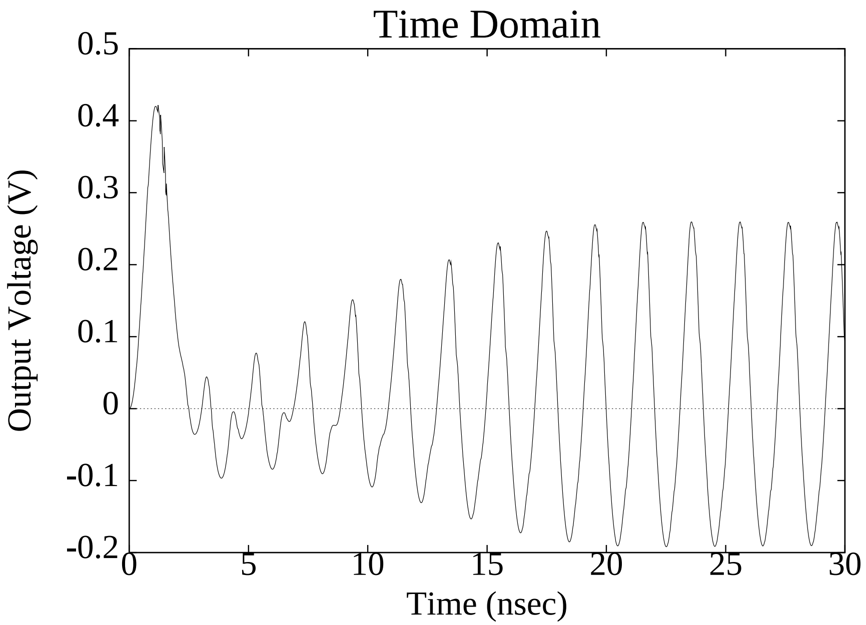
$$g = \dot{I}_{ds} + \left(\dot{C}_{gs} + G_{gs}\right)\dot{u}_2 + \dot{G}_{gs}u_2 - \frac{\dot{u}_3 - \dot{v}_A}{R_A} - \frac{u_3}{L_A}$$

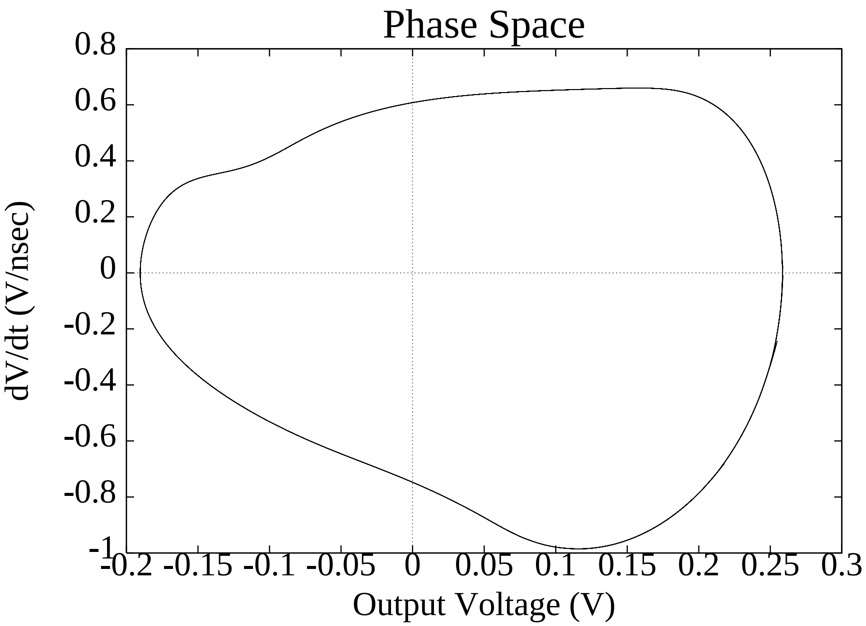
$$h = -\frac{1}{L_2}\left(u_2 + u_3 + v_s\right) - \left(\dot{C}_{gd} + G_{gd}\right)\left(\dot{u}_2 - \dot{u}_1\right) - \dot{G}_{gd}\left(u_2 - u_1\right) - \left(\dot{C}_{gs} + G_{gs}\right)\dot{u}_2 - \dot{G}_{gd}u_2$$

$$k_1 = C_{gd} + C_{gs},\ k_2 = C_{gs}C_{gd} + C_{ds}C_{gd} + C_{gs}C_{ds}$$

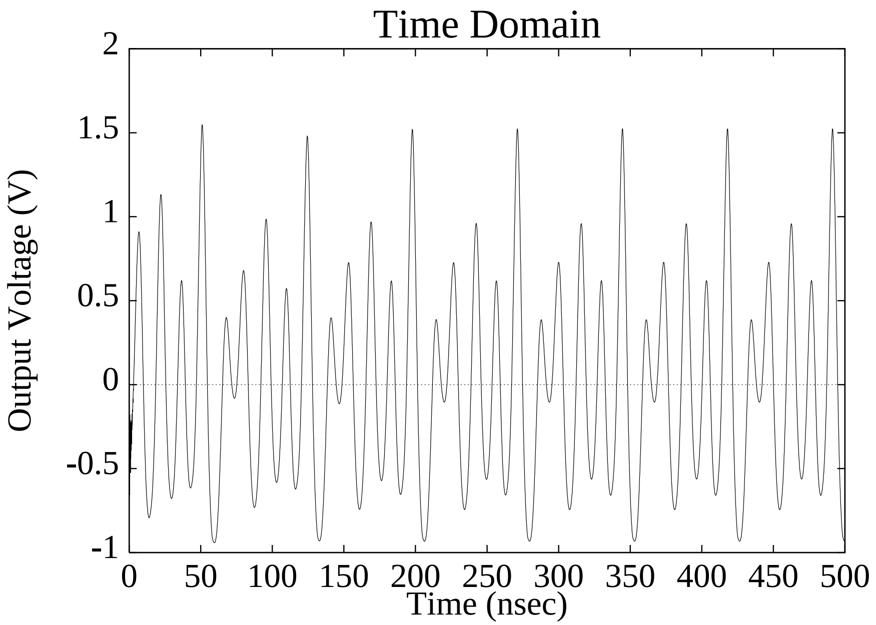
$$h' = \frac{h}{k_1}$$

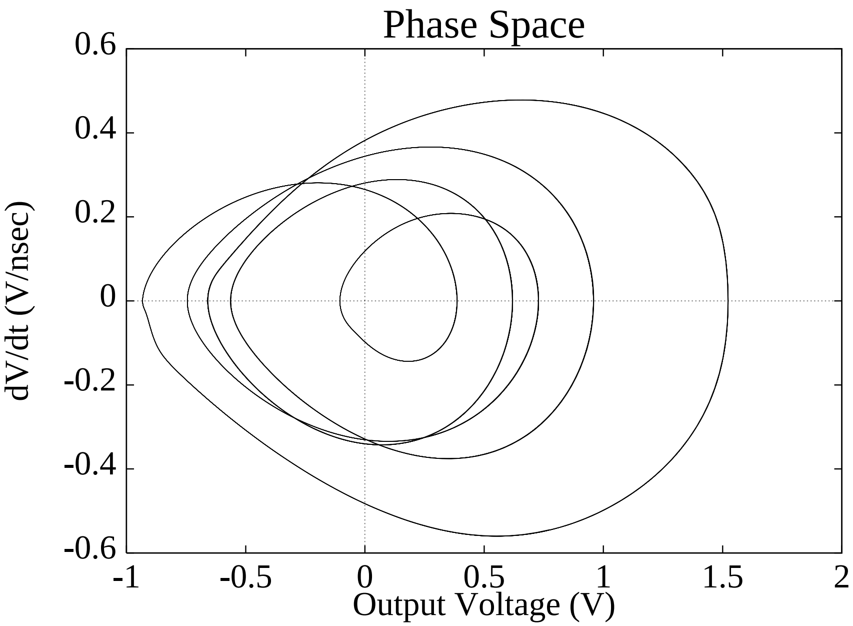
Free-Running Oscillation



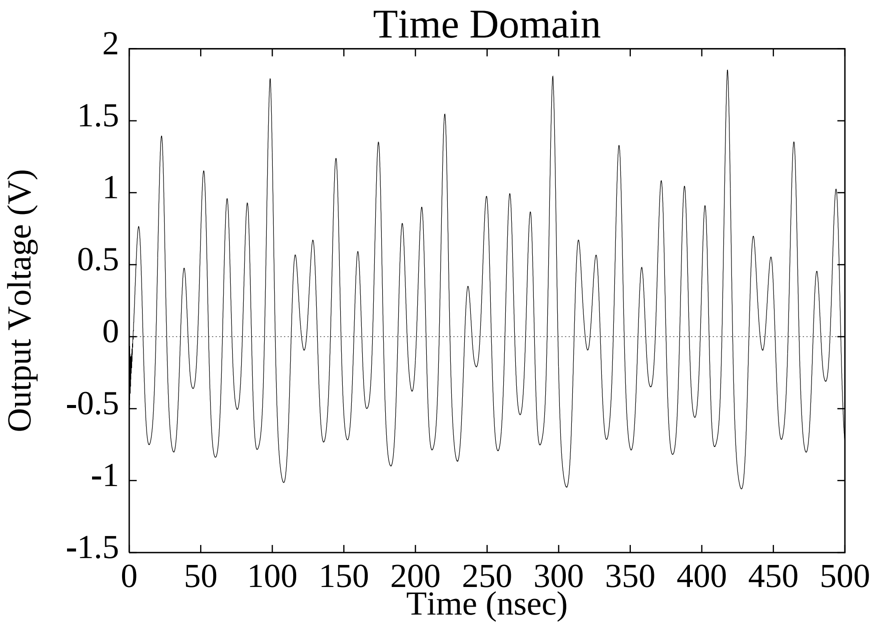


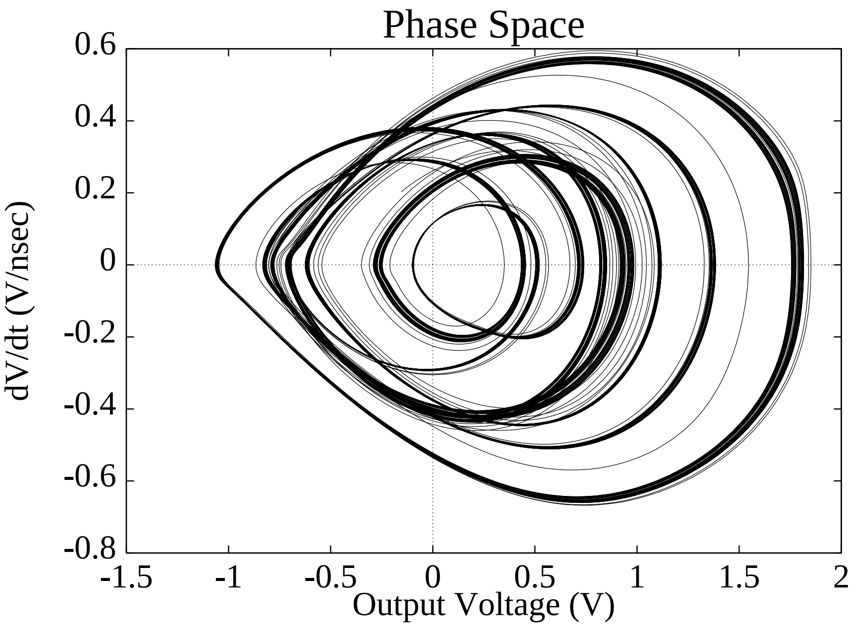
Free-Running Five-Cycle Oscillation





Free-Running Possible Chaotic Oscillation

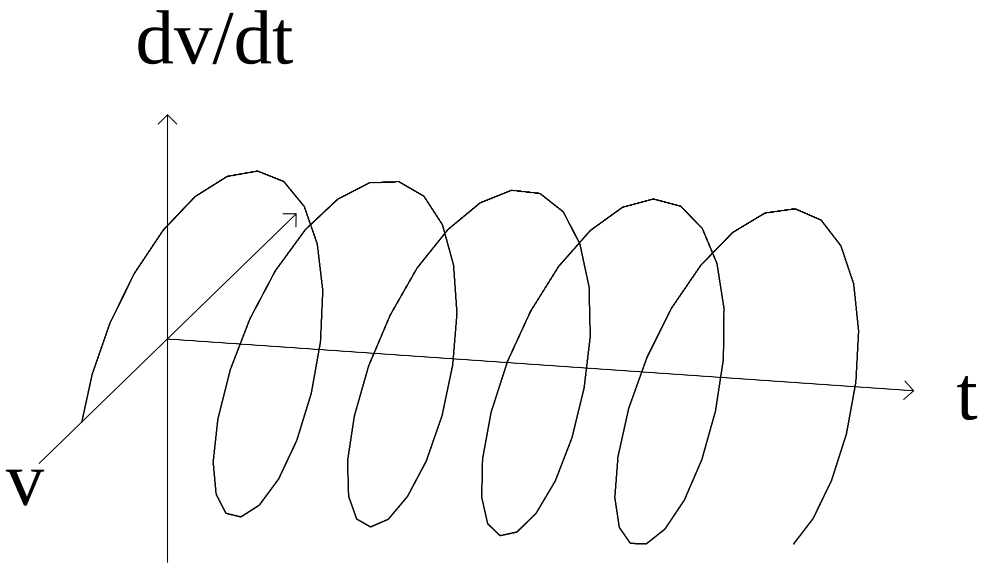




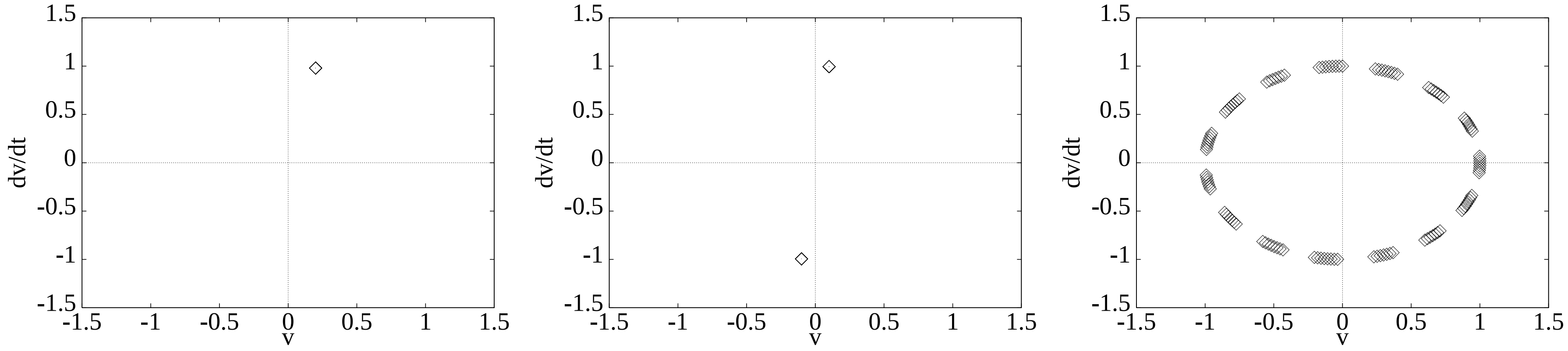
Injection Locking

- Determination of injection locked range
- Look at large variations in f_{inj}, P_{inj}
- Investigate details of some regions of (f_{inj}, P_{inj})
- Compare with experimental results on a 500 MHz lumped-element NEC 71084 MESFET oscillator circuit

Poincare Sections



Purpose : Reduce dimension of data to facilitate analysis

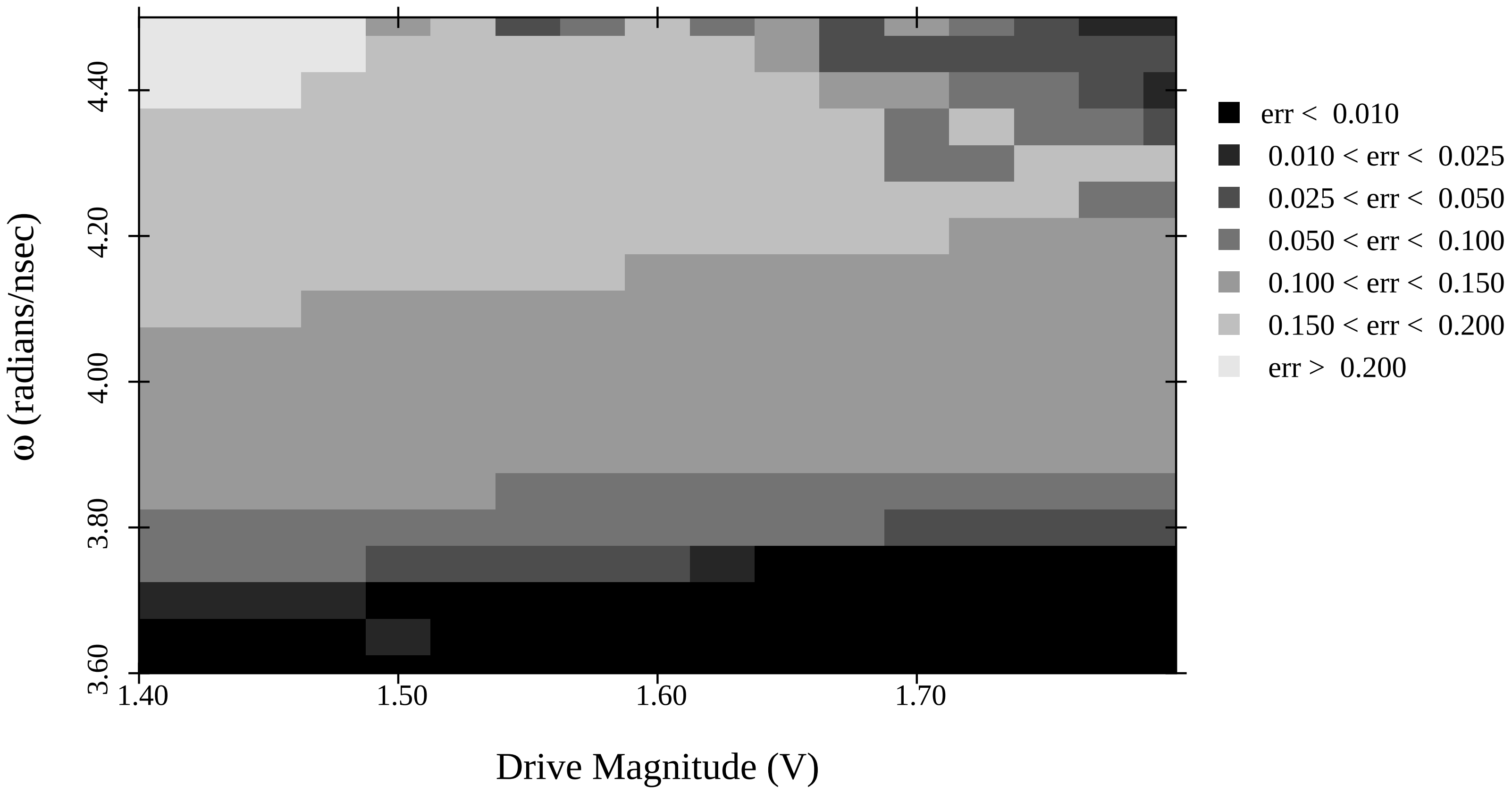


Lock Determination

$$error_n = error_{n-1} + \frac{1}{3}\Big[(v_n - v_{n-i})^2 + (\dot{v}_n - \dot{v}_{n-i})^2 + (v_n - v_{n-2i})^2 + (\dot{v}_n - \dot{v}_{n-2i})^2 + (v_n - v_{n-3i})^2 + (\dot{v}_n - \dot{v}_{n-3i})^2 \Big]^{1/2}$$

- Linear interpolation used to estimate exact values for v and $\dot{v}$
- Error calculated per cycle to reduce differences due to drive periods
- Sectioning done when drive signal crosses 0 V in a positive direction

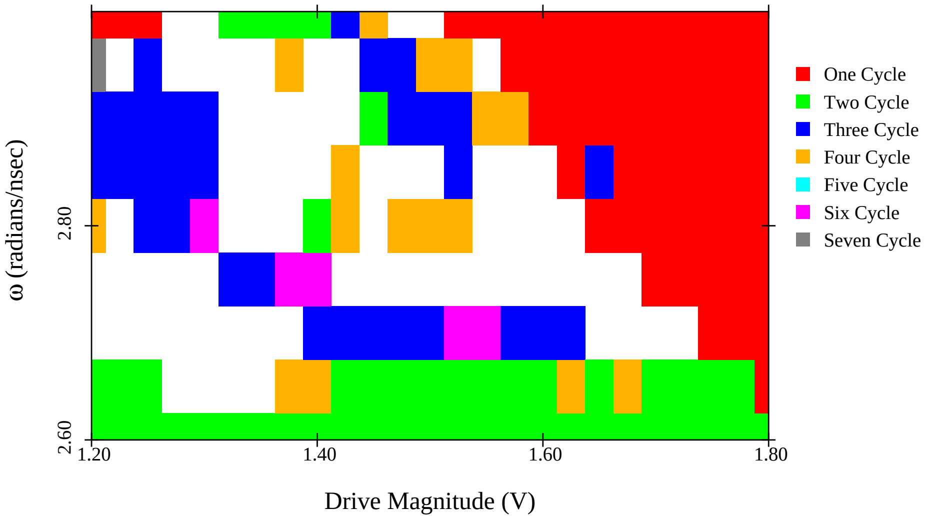
Cutoff Choice

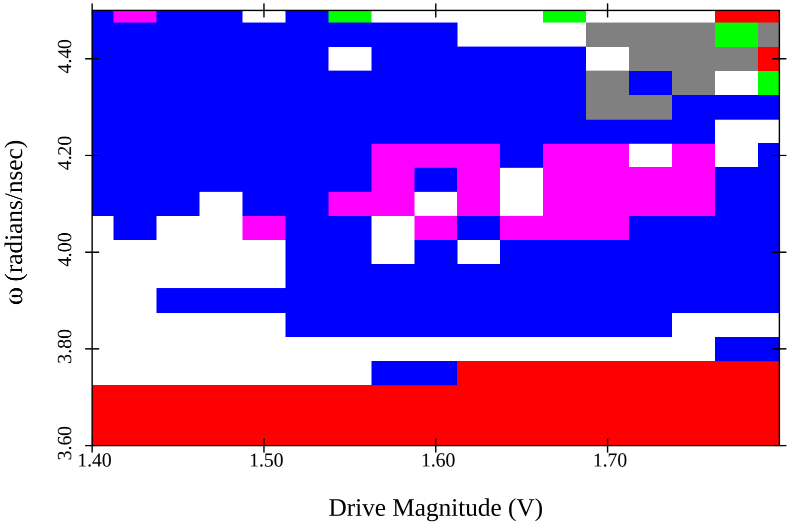


Injection-Locking Scan

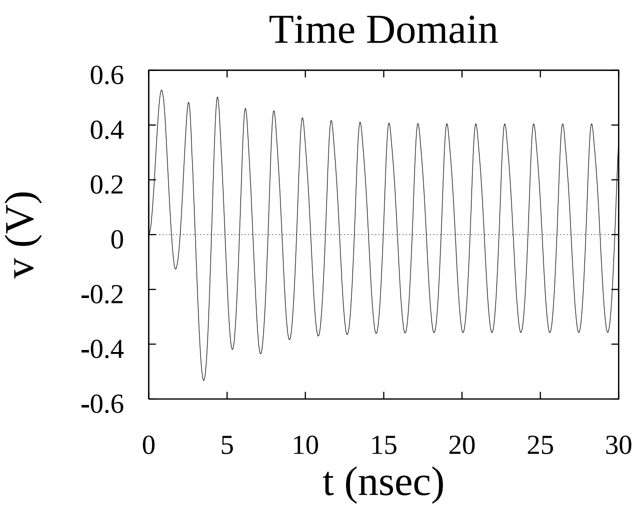
Use color slide

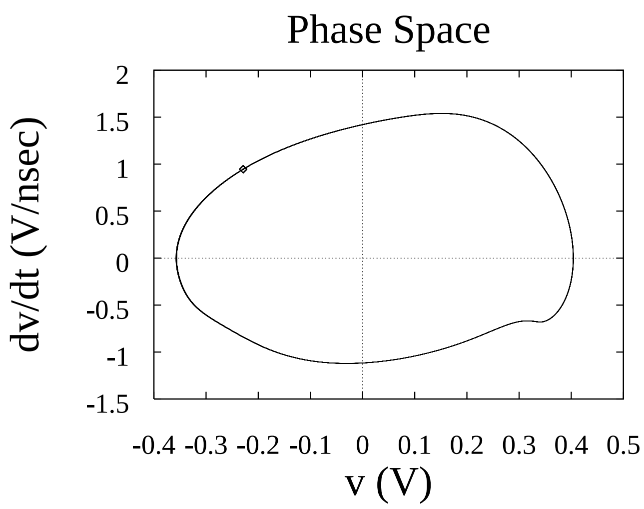
Detailed Areas

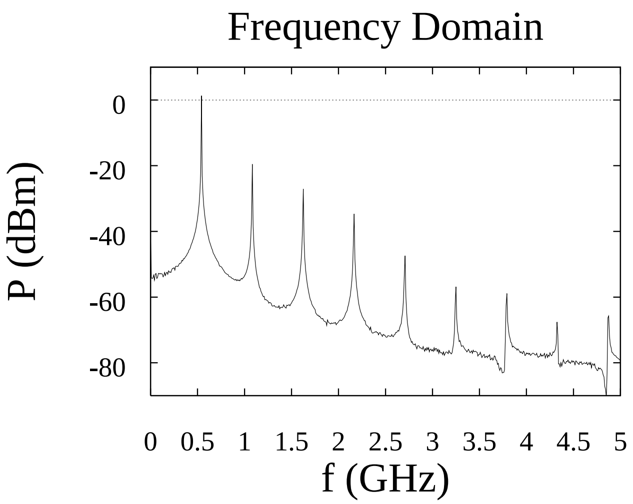




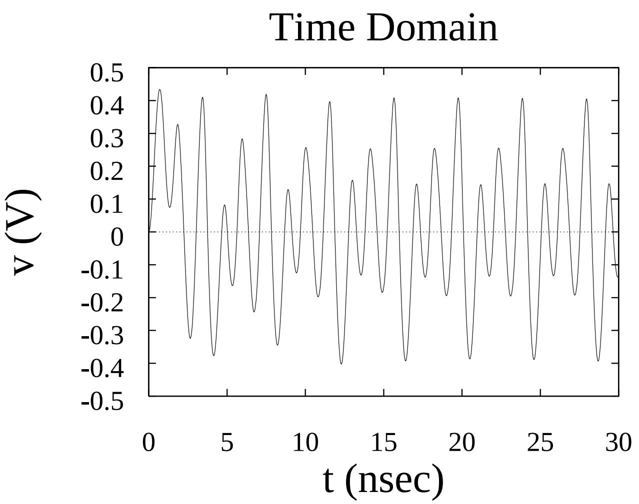
Driven Locked Oscillation

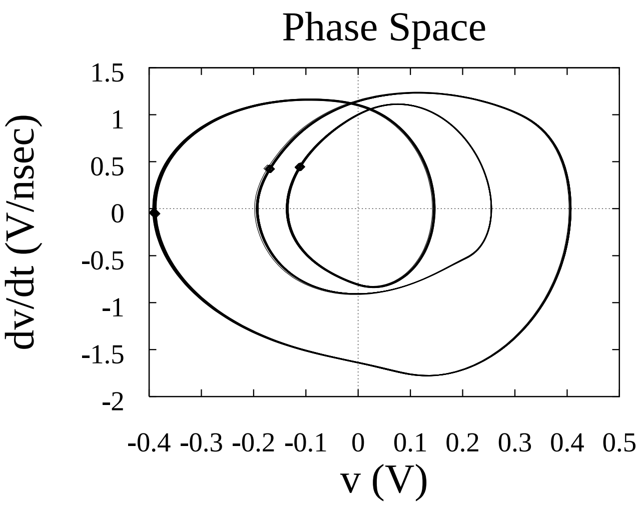


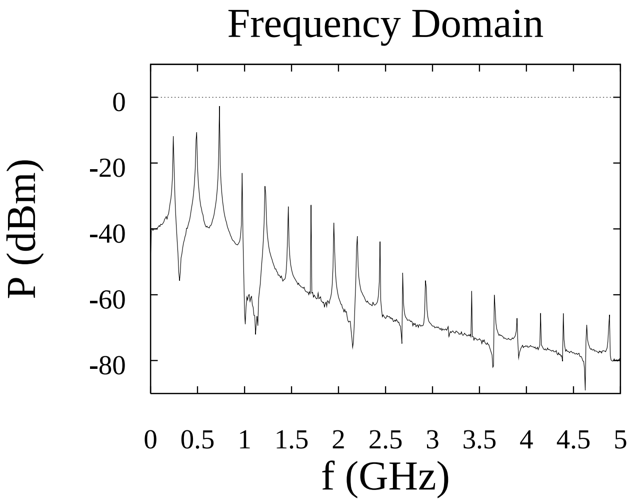




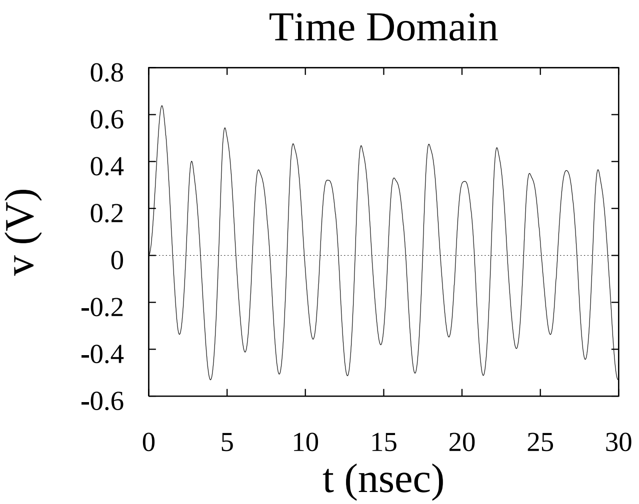
Driven Three-Cycle Oscillation

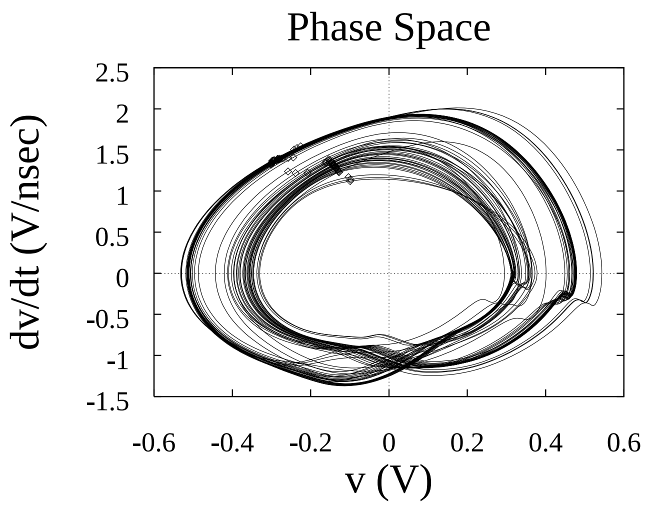


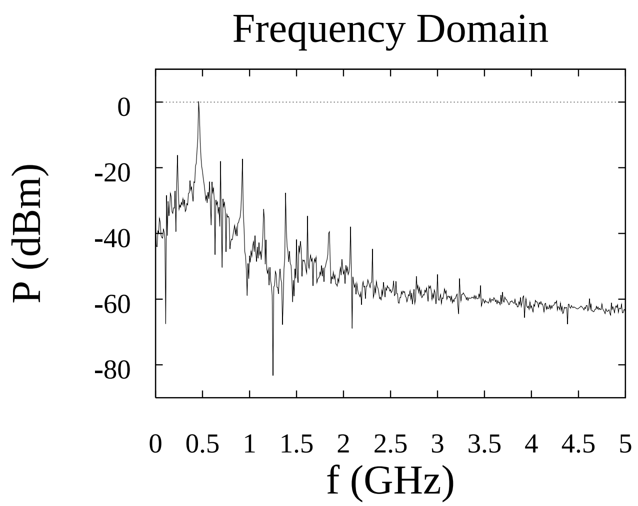




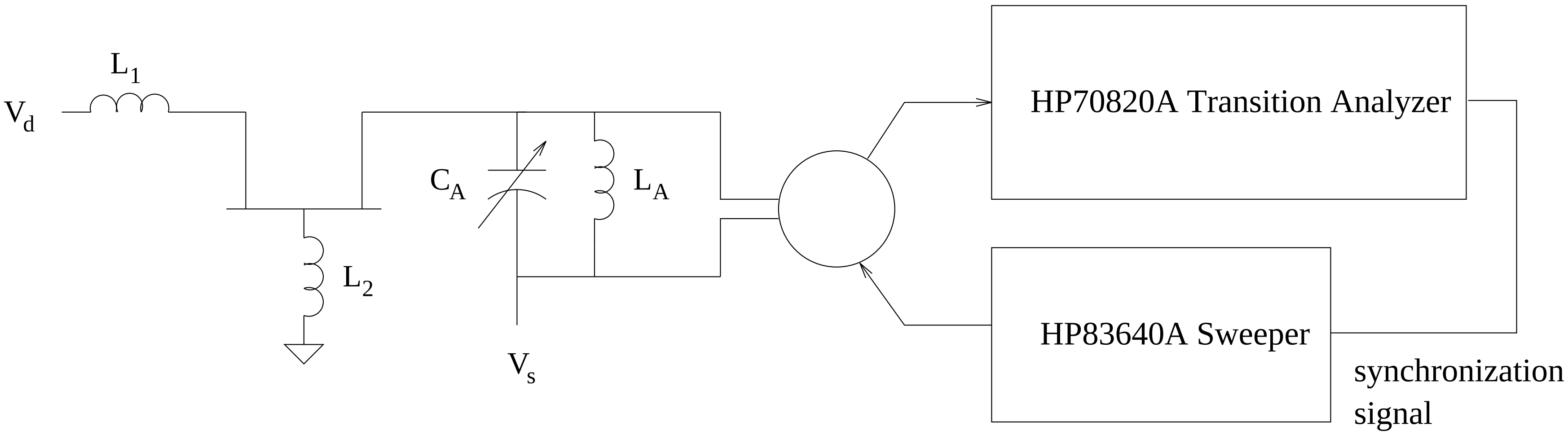
Driven Apparent Chaos







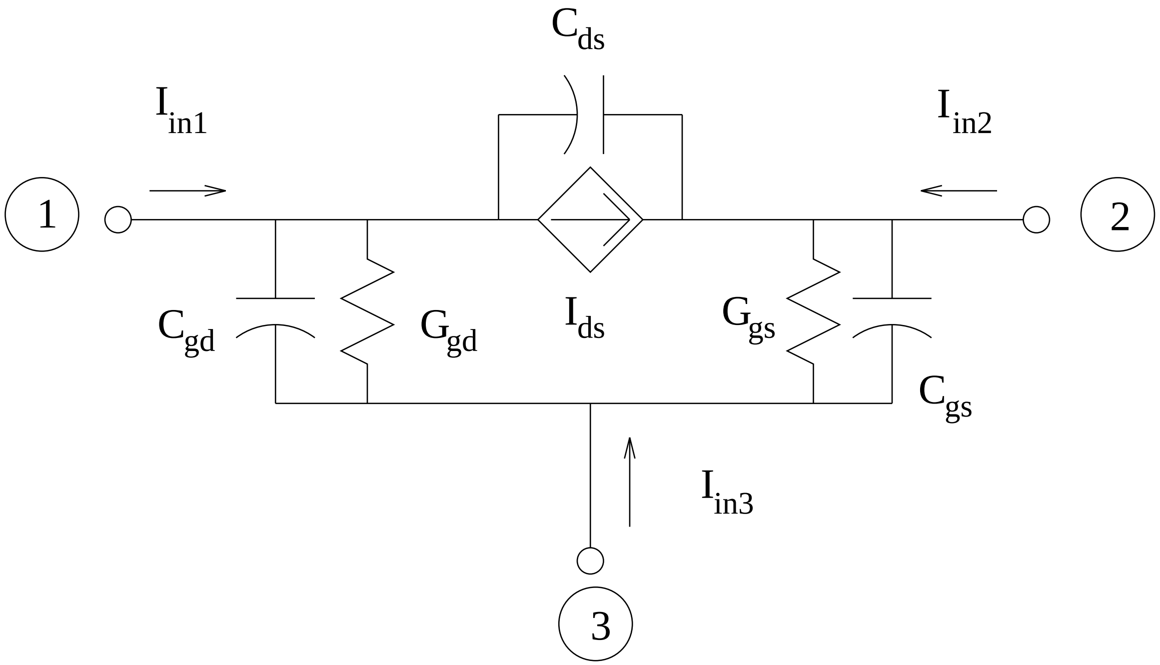
Experimental Design



Limitations of Method

- Limited to single circuit geometry
- Uses lumped external elements
- More representative of RF than microwave circuits

General Circuit



General Equations

$$\left.\begin{array}{rcl} \dot{u}_1 & = & u_3 \\ \dot{u}_2 & = & u_4 \\ \dot{u}_3 & = & g' + \dfrac{h'}{k_1} \\ \dot{u}_4 & = & g'k_2 + h'\left(1 + \dfrac{k_2}{k_1}\right) \end{array}\right\}$$

where

$$u_1 = v_1 - v_2 = v_{ds},\ u_2 = v_3 - v_2 = v_{gs},\ u_3 = \dot{u}_1,\ u_4 = \dot{u}_2$$

and

$$g = \dot{I}_{in1} - (\dot{u}_1 - \dot{u}_2)\left(\dot{C}_{gd} + G_{gd}\right) - \dot{G}_{gd}(u_1 - u_2) - \dot{I}_{ds}$$

$$h = \dot{I}_{in3} - (\dot{u}_2 - \dot{u}_1)\left(\dot{C}_{gd} + G_{gd}\right) - \dot{G}_{gd}(u_2 - u_1) - \dot{u}_2\left(\dot{C}_{gs} + G_{gs}\right) - u_2\dot{G}_{gs}$$

$$k_1 = 1 + \tfrac{C_{ds}}{C_{gd}} - k_2,\ k_2 = \tfrac{C_{gd}}{C_{gs}+C_{gd}}$$

$$g' = \tfrac{gC_{gd}}{k_1},\ h' = \tfrac{h}{C_{gd}+C_{gs}}$$

and subject to the constraint

$$\dot{I}_{in1} + \dot{I}_{in2} + \dot{I}_{in3} = 0$$

Characteristics of the General Formulation

- Requires $\dot{I}_{in1}$ and $\dot{I}_{in3}$ as inputs
- Gives $\dot{I}_{in2}$, u_1, u_2, $\dot{u}_1$, and $\dot{u}_2$ as outputs
- More flexible circuit model implementations
- Can be incorporated into a variety of circuit solvers
- Can also be reformulated without current derivatives

Future Work

- Finish experimental verification of injection-locking
- Extend source-load model to include mutual coupling between 2 or more oscillators
- Implement general model
- Explore effects of parameter changes on locking ability